

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

JEE Main (Archive)

Daily Tutorial Sheet 1

1.(C) Given, α and β are the roots of the equation $x^2 - 6x - 2 = 0$.

$$\therefore a_n = \alpha^n - \beta^n \text{ for } n \geq 1$$

$$a_{10} = \alpha^{10} - \beta^{10}$$

$$a_8 = \alpha^8 - \beta^8$$

$$a_9 = \alpha^9 - \beta^9$$

$$\text{Now, consider } \frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)} = \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} = \frac{6}{2} = 3$$

$$\left[\begin{array}{l} \because \alpha \text{ and } \beta \text{ are the roots of } x^2 - 6x - 2 = 0 \text{ or } x^2 = 6x + 2 \\ \Rightarrow \alpha^2 = 6\alpha + 2 \Rightarrow \alpha^2 - 2 = 6\alpha \text{ and } \beta^2 = 6\beta + 2 \Rightarrow \beta^2 - 2 = 6\beta \end{array} \right]$$

Alternate Solution

Since, α and β are the roots of the equation $x^2 - 6x - 2 = 0$.

$$\text{or } x^2 = 6x + 2$$

$$\therefore \alpha^2 = 6\alpha + 2 \Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8 \quad \dots (i)$$

$$\text{Similarly, } \beta^{10} = 6\beta^9 + 2\beta^8 \quad \dots (ii)$$

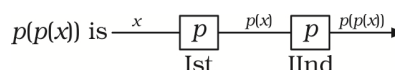
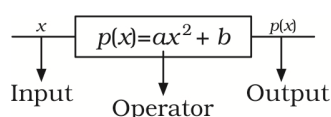
On subtracting Eq. (ii) from Eq. (i), we get

$$\alpha^{10} - \beta^{10} = 6(\alpha^9 - \beta^9) + 2(\alpha^8 - \beta^8) \quad (\because a_n = \alpha^n - \beta^n)$$

$$\Rightarrow a_{10} = 6a_9 + 2a_8 \Rightarrow a_{10} - 2a_8 = 6a_9 \Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$$

2.(D) If quadratic equation has purely imaginary roots, then coefficient of x must be equal to zero.

Let $p(x) = ax^2 + b$ with a, b of same sign and $a, b \in R$.



$$\text{Roots of } p(x) \text{ are } \pm i\sqrt{\frac{b}{a}} \quad \text{Thus, } p(p(x)) = 0 \text{ iff } p(x) = \pm i\sqrt{\frac{b}{a}}$$

$$\text{Iff } ax^2 + b = \pm i\sqrt{\frac{b}{a}}$$

$$\text{Iff } x^2 = \frac{1}{a} \left(-b \pm i\sqrt{\frac{b}{a}} \right)$$

Clearly $x \notin R$ Also, x can't be purely imaginary because then $x^2 \in R^-$

Hence, Option (D)

3.(B) Given, $x^2 - |x + 2| + x > 0 \quad \dots (i)$

Case I When $x + 2 \geq 0$

$$\therefore x^2 - x - 2 + x > 0 \Rightarrow x^2 - 2 > 0$$

$$\Rightarrow x < -\sqrt{2} \text{ or } x > \sqrt{2} \Rightarrow x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty) \quad \dots (ii)$$

Case II When $x + 2 < 0$

$$\therefore x^2 + x + 2 + x > 0 \Rightarrow x^2 + 2x + 2 > 0 \Rightarrow (x+1)^2 + 1 > 0$$

Which is true for all x .

$$\therefore x \in (-\infty, -2) \quad \dots (iii)$$

From Eqs. (ii) and (iii), we get

$$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, -\infty)$$

4.(B) Given, $\log_4(x-1) = \log_2(x-3) = \log_{4^{1/2}}(x-3)$

$$\Rightarrow \log_4(x-1) = 2\log_4(x-3) \Rightarrow \log_4(x-1) = \log_4(x-3)^2$$

$$\Rightarrow (x-3)^2 = x-1 \Rightarrow x^2 + 9 - 6x = x-1 \Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2, \text{ or } x = 5$$

$$\Rightarrow x = 5 \quad [\because x = 2 \text{ makes } \log(x-3) \text{ undefined}]$$

Hence, one solution exists.

5.(C) Let α, α^2 be the roots of $3x^2 + px + 3 = 0$

$$\text{Now, } S = \alpha + \alpha^2 = -p/3,$$

$$\alpha^3 = 1 \Rightarrow \alpha = 1, \omega, \omega^2$$

$$\alpha \neq 1 \text{ because } -p/3 = \alpha + \alpha^2 = 2 \Rightarrow p < 0 \text{ (contradiction)}$$

$$\text{Now, } \alpha + \alpha^2 = -p/3$$

$$\Rightarrow \omega + \omega^2 = -p/3 \Rightarrow -1 = -p/3 \Rightarrow p = 3$$

6.(A) Since, $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1} \Rightarrow (x+1) + (x-1) - 2\sqrt{x^2-1} = 4x-1$

$$\Rightarrow 1-2x = 2\sqrt{x^2-1} \Rightarrow 1+4x^2-4x = 4x^2-4 \Rightarrow 4x-5=0 \Rightarrow x = \frac{5}{4}$$

But it does not satisfy the given equation.

Hence, no solution exists.

7.(ABC) Given, $\frac{3}{x^4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \sqrt{2} \Rightarrow \frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \log_x \sqrt{2}$

$$\Rightarrow \frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \frac{1}{2\log_2 x} \Rightarrow 3(\log_2 x)^3 + 4(\log_2 x)^2 - 5(\log_2 x) - 2 = 0$$

$$\text{Put } \log_2 x = y$$

$$\therefore 3y^3 + 4y^2 - 5y - 2 = 0 \Rightarrow (y-1)(y+2)(3y+1) = 0$$

$$\Rightarrow y = 1, -2, -1/3 \Rightarrow \log_2 x = 1, -2, -1/3$$

$$x = 2, \frac{1}{2^{1/3}}, \frac{1}{4}$$

8.(A) Given, $x - \frac{2}{x-1} = 1 - \frac{2}{x-1} \Rightarrow x = 1$

But at $x = 1$, the given equation is not defined.

Hence, no solution exists.

9.(D) Let $y = \frac{x^2 - (a+b)x + ab}{x-c}$

$$\Rightarrow yx - cy = x^2 - (a+b)x + ab \Rightarrow x^2 - (a+b+y)x + (ab+cy) = 0$$

For real roots, $D \geq 0$

$$\Rightarrow (a+b+y)^2 - 4(ab+cy) \geq 0 \Rightarrow (a+b)^2 + y^2 + 2(a+b)y - 4ab - 4cy \geq 0$$

$$\Rightarrow y^2 + 2(a+b-2c)y + (a-b)^2 \geq 0$$

Which is true for all real values of y .

$$\therefore D \leq 0$$

$$4(a+b-2c)^2 - 4(a-b)^2 \leq 0$$

$$\Rightarrow 4(a+b-2c+a-b)(a+b-2c-a+b) \leq 0 \Rightarrow (2a-2c)(2b-2c) \leq 0$$

$$\Rightarrow (a-c)(b-c) \leq 0 \Rightarrow (c-a)(c-b) \leq 0$$

$$\Rightarrow c \text{ must lie between } a \text{ and } b$$

$$\text{i.e., } a \leq c \leq b \text{ or } b \leq c \leq a$$

10.(A) Since, $|x|^2 - 3|x| + 2 = 0$

$$\Rightarrow (|x|-1)(|x|-2) = 0 \Rightarrow |x| = 1, 2 \therefore x = 1, -1, 2, -2$$

Hence, four real solutions exist.